TKN/KS/16/5883

Bachelor of Science (B.Sc.) Semester—V
(C.B.S.) Examination
MATHEMATICS
Paper—I
(M_9-Analysis)

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Show that the Fourier Series for the periodic function defined by $f(x) = 0, -\pi \le x < 0$ and $f(x) = x^2, 0 \le x < \pi$ is :

$$f(x) = \frac{\pi^2}{6} + 2\sum_{1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$$

$$+ \pi \sum_{1}^{\infty} \frac{(-1)^{n+1} \sin nx}{n} - \frac{4}{\pi} \sum_{1}^{\infty} \frac{\sin(2n-1)x}{(2n-1)^3}.$$

6

(C) If $F \in \mathbb{R}(\alpha)$ and $g \in \mathbb{R}(\alpha)$ on [a, b], then prove that :

(i) $f_g \in \mathbb{R}(\alpha)$

(ii) $|f| \in \mathbb{R}(\alpha)$ and $\left|\int_a^b f d\alpha\right| \le \int_a^b |f| d\alpha$. 6

(D) Suppose F and G are differentiable functions on [a, b], $F' = f \in \mathbb{R}$ and $G' = g \in \mathbb{R}$. then prove that :

$$\int_a^b F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx.$$

6

UNIT—III

3. (A) If f(z) = u + iv is an analytic function in a domain D, prove that the curves u = constant, v = constant form two orthogonal families. 6

(B) If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine v so that u + iv is an analytic function of x + iy. 6

OR

(C) If f(z) is an analytic function with constant modulus, then prove that it is constant. 6

(D) If $u = x^2 - y^2$, $v = -\frac{y}{(x^2+y^2)}$, then show that both u and v satisty Laplace's equation, but u + iv is not an analytic function of z. 6

UNIT—IV

4. (A) Let a rectangular domain R be bounded by x = 0, y = 0, x = 2, y = 1 in z-plane. Determine the region R' of w-plane into which R is mapped under the transformation w = z + (1 – 2i). 6

(B) Prove that the cross ratio remains invariant under a Bilinear Transformation. 6

OR

(C) Find the bilinear transformation which maps the points z = –2, 0, 2 into the point w = 0, i, – i respectively. 6

(D) Establish the relation $w = \frac{iz+2}{4z+i}$ transforms the real axis in z-plane to a circle in the w-plane. Find the centre and the radius of the circle and the point in the z-plane which is mapped on the centre of the circle. 6

5. (A) If $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then prove that $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx;\ n = 1, 2, 3, \ldots$ 1½

(B) Find the Fourier coefficient b_n for the function f(x) = x, $-\pi \le x \le \pi$. 1½

(C) For any partition P of [a, b], prove that $L(P, f) \le U(P, f)$. 1½

(B) Let f(x) be an integrable function defined on the interval $-\pi \le x \le \pi$. If f(x) is odd, then prove that its Fourier Series has only sine terms and the coefficients are given by :

$$a_n = 0,\ n = 0, 1, 2, 3, \ldots\ldots$$

and $$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx,\ n = 1, 2, 3, \ldots.$$ 6

OR

(C) Show that the cosine series for x^2 is :

$$x^2 = \frac{\pi^2}{3} + 4\sum_{1}^{\infty}\frac{(-1)^n \cos nx}{n^2};\quad -\pi \le x \le \pi.$$ 6

(D) Expand f(x) in a Fourier Series on the interval $-2 \le x < 2$ if $f(x) = 0$ for $-2 \le x < 0$ and $f(x) = 1$ for $0 \le x < 2$. 6

UNIT—II

2. (A) Prove that $f \in \mathbb{R}(\alpha)$ or [a, b] if and only if for every $\in > 0$ there exists a partition P such that :

$U(P, f, \alpha) - L(P, f, \alpha) < \in$. 6

(B) Let $f \in \mathbb{R}$ on [a, b]. For $a \le x \le b$, put $F(x) = \int_a^x f(t)\,dt$, where F is continuous on [a, b].

If f is continuous at a point x_0 of [a, b], then prove that F is differentiable at x_0 and $F'(x_0) = f(x_0)$. 6

OR

(D) If $f \in \mathbb{R}(\alpha)$ on [a, b], then prove that $cf \in \mathbb{R}(\alpha)$ and $\int_a^b cf\,d\alpha = c\int_a^b f\,d\alpha$ for every constant c. 1½

(E) If $w = f(z) = u + iv$ be an analytic function in a domain D, then prove that $\frac{dw}{dz} = \frac{\partial w}{\partial x}$. 1½

(F) Prove that $u = y^3 - 3x^2y$ is a harmonic function. 1½

(G) Find the fixed points of the bilinear transformation $w = \frac{z-1}{z+1}$. 1½

(H) Show that $w = iz + i$ maps half plane $x > 0$ onto half plane $v > 1$. 1½